

SWR and Smith Chart Animations
by Rick Nungester WA6NDR, 12/10/2020

Summary: Computer animations demonstrate incident and reflected waveforms, standing waves, standing wave ratio, complex reflection coefficient, impedance, admittance, and the Smith Chart. Emphasis is on concepts and images more than the underlying math. (The first 3 images in this handout are still screenshots of the moving animations used during the talk.)

- "Phasor" Geometer's Sketchpad file
 - A phasor is a graphical way to show the relationship between 2 sine waves of the same frequency. A point on a plane defines their relative amplitude and phase. The two sine waves can be 2 voltages, a voltage and a current, waves on a string, water waves, whatever.
 - Show: Same phase, different amplitude; same amplitude, different phase (0, +/-90, 180 degrees, lead/lag); arbitrary amplitude and phase (I/Q modulation, capacitor/inductor voltage/current, ELI the ICE man). A very flexible and useful concept.

- "Smith_VSWR" Geometer's Sketchpad file
 - Incident wave; reflected wave; incident + reflected = standing; peaks and valleys; voltage standing wave ratio; peaks (or valleys) are separated by wavelength/2 on the coax. Example: 2-meter band; peaks are separated by about 1 meter.
 - * KEY IDEA: The Smith Chart is a phasor, relating the reflected wave to the incident wave. A point on the Smith Chart is called "Gamma" or the "Complex Reflection Coefficient" or "S11".
 - "Reflection Coefficient" (no "Complex") is the distance from the center of the circle to Gamma, with value 0 to 1. It is also called "rho" (Greek letter) or "magnitude of the Gamma". Knowing it allows calculating:
 - VSWR
 - Reflected Power (%)
 - Return Loss (dB)
 - Transmitted Power (%)
 - Mismatch Loss (dB)
 - Gamma is a "complex number" or "vector" (as in Vector Network Analyzer). Your scientific calculator likely has complex operations. Knowing Gamma allows calculating:
 - Impedance $Z = \text{resistance } R + j * \text{reactance } X$
 - Admittance $Y = G + j * B = \text{conductance} + j * \text{susceptance}$
 - "j" is the square root of -1, used in complex math.
 - Move point Gm1 (Gammal) to key points:
 - Short (reflected amplitude = 1, phase = 180 degrees).
 - Open (reflected amplitude = 1, phase = 0 degrees).
 - 50 Ohm Load (reflected amplitude = 0, usually the goal).
 - 33 Ohm Load (SWR = 1.5).
 - 75 Ohm Load (same distance from center, SWR = 1.5).
 - Top half-circle = inductor. A small (large) inductor looks like a short (open) circuit.
 - Bottom half-circle = capacitor. A small (large) capacitor looks like an open (short) circuit.
 - Moving the measurement point back away from the antenna changes only the angle of Gamma clockwise by 360 degrees per 1/2 wavelength. Show Gm2 is always the same SWR as Gm1 but with different angle.

- "Smith_G_Z_Y" Geometer's Sketchpad file
 - This shows a Smith Chart (radius = 1) on top of an XY graph showing normalized impedance (Zn) and admittance (Yn). "Normalized" means $Z0 = 1$ ohm instead of the usual 50 ohms. It makes the graphing work.

- Each of the 3 points are phasors, relating same-frequency sine waves:
 - Gamma relates reflected voltage to incident voltage.
 - Impedance relates voltage to current.
 - Admittance relate current to voltage.
- The operations +, -, *, / are complex (vector) operations here. Once that math is understood, the formulas are fairly simple. The math just maps point G_m to points Z_n and Y_n .
- Rotate G_m around the origin (a good load). See Z_n and Y_n rotate around 1 ($Z = Z_0$, $Y = Y_0$, also a good load).
- G_m right, to open ($Z_n \rightarrow \text{infinity}$, $Y_n \rightarrow 0$).
- G_m left, to short ($Z_n \rightarrow 0$, $Y_n \rightarrow \text{infinity}$).
- Notation: $|x|$ means "magnitude of the complex number x ", or how far it is from the center of the circle.
- G_m on the vertical axis ($|Z_n| = |Y_n| = 1$, but opposite phase).

- Advanced Topics
 - Everything presented so far applies to a single frequency. Antenna Analyzers and Vector Network Analyzers sweep frequency through a range.
 - "RF Demo Kit" (for NanoVNA) PC board Smith Chart images:
 - Resistance only, no reactance, so no change with frequency: 13 (Short); 14 (Open); 15 (50 ohms); 3 (33 ohms); 4 (75 Ohms).
 - 9 (capacitor, low frequency = open, high frequency = short).
 - 10 (inductor, low frequency = short, high frequency = open).
 - 7 (RC Circuit, low frequency = open, high frequency = 50 ohm load).
 - "Smith Chart Calc" Android app image, showing 9 possible components and how they move Gamma on the Smith Chart.
 - Series/shunt capacitor
 - Series/shunt inductor
 - Series/shunt resistor
 - Shunt open coax line
 - Series coax line
 - Shunt short coax line
 - 2-port measurements: amplifiers, attenuators, filters, delay lines, antenna tuners, etc. These four S-Parameters are just points on a phasor (complex numbers), relating two measured voltages.
 - S_{11} : Port 1 reflected to port 1 incident (Gamma).
 - S_{21} : Port 2 output to port 1 input (gain, attenuation, delay).
 - S_{12} : Port 1 output to port 2 input (reverse isolation).
 - S_{22} : Port 2 reflected to port 2 incident (\rightarrow output impedance).
 - Nothing is perfect.
 - The output impedance of your transmitter is not 50 ohms. The reflected wave hits the transmitter and re-reflects to add to the incident wave.
 - Directional couplers are imperfect. Some of the incident wave is present in the measurement of the reflected wave.

- References
 - https://en.wikipedia.org/wiki/Smith_chart
 - <https://en.wikipedia.org/wiki/Impedance>
 - <https://en.wikipedia.org/wiki/Admittance>
 - https://en.wikipedia.org/wiki/Standing_wave_ratio
 - <https://www.dynamicgeometry.com> (The Geometer's Sketchpad software is no longer supported, but is still available on eBay.)

- Questions? Related comments or experiences?

Phasor

Move point R to set the amplitude and phase of the Relative Sine Wave.

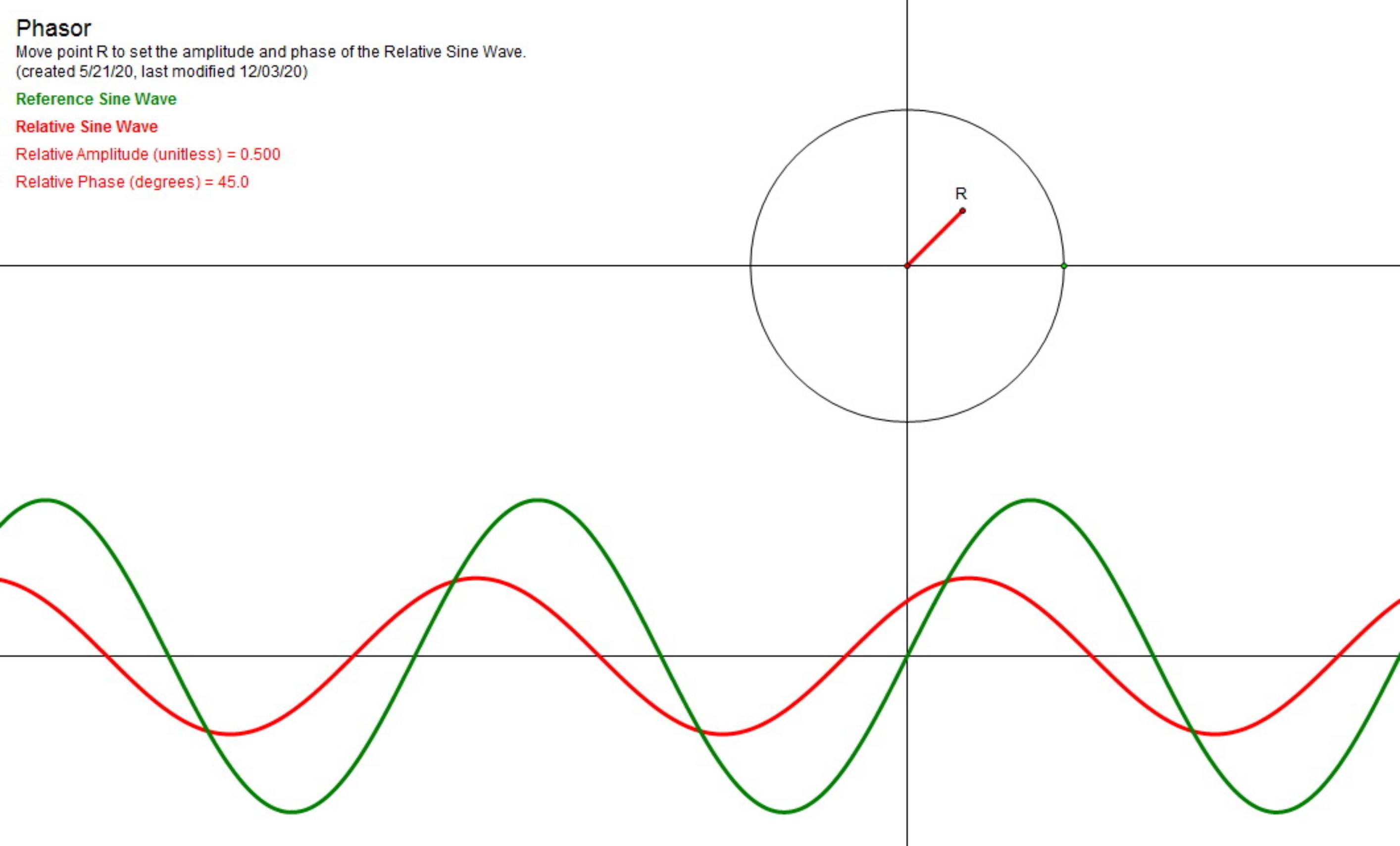
(created 5/21/20, last modified 12/03/20)

Reference Sine Wave

Relative Sine Wave

Relative Amplitude (unitless) = 0.500

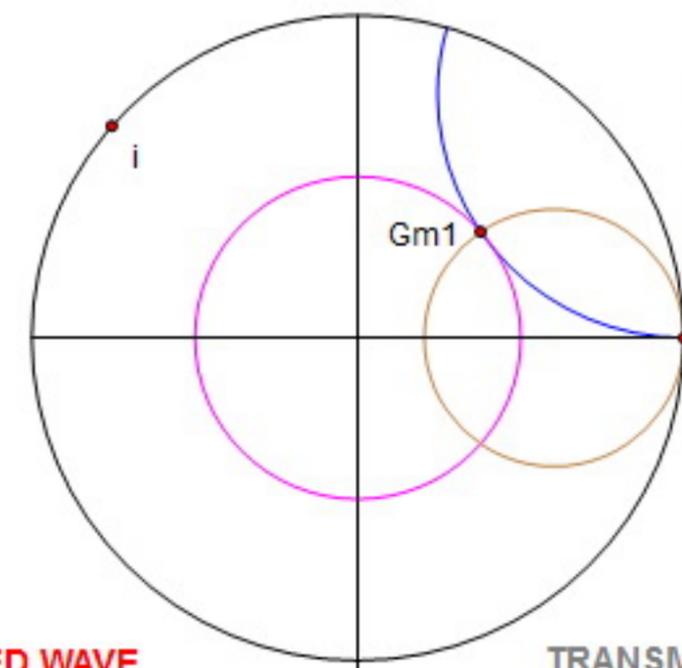
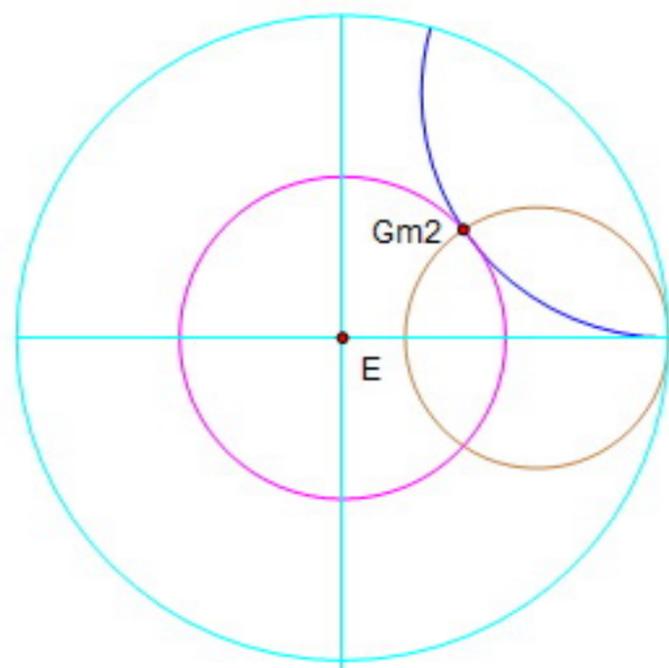
Relative Phase (degrees) = 45.0



Smith Chart & VSWR

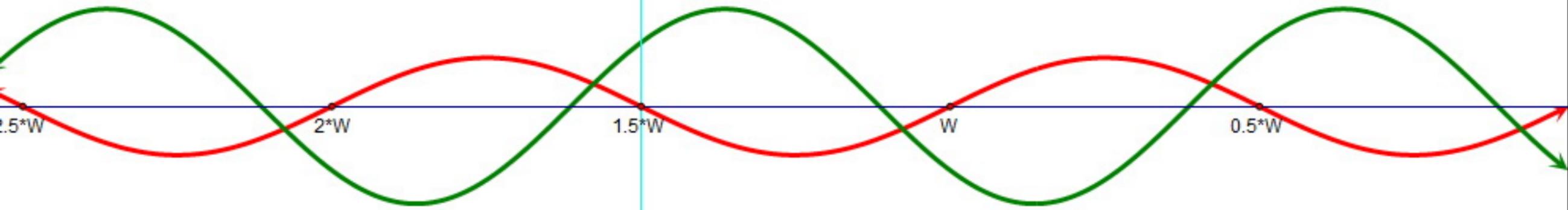
Move point Gm1 (Gamma1) on the Smith Chart.
Move point E on the coax.

Demo Notes: Select i. Animate Point (CCW).
Hide i. Close Motion Controller. Hide this note.
(created 10/9/20, last modified 12/03/20)



Smith Chart
Z0 (Ohms) = 50.00
R (Ohms) = 75.54
X (Ohms) = 66.16
VSWR = 3.00

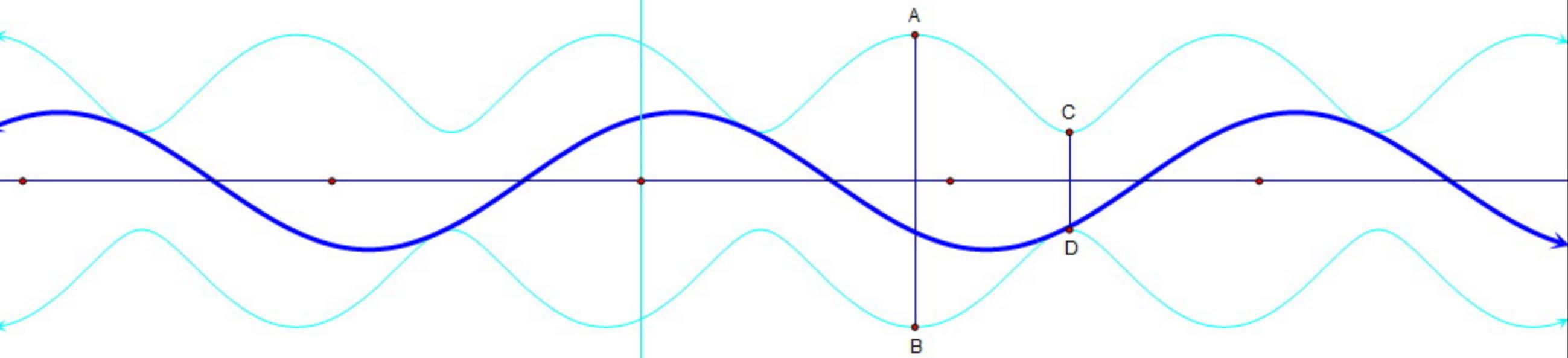
INCIDENT WAVE



REFLECTED WAVE
Reflected Power (%) = 25.0
Return Loss (dB) = 6.02

TRANSMITTED WAVE
Transmitted Power (%) = 75.0
Mismatch Loss (dB) = 1.25

INCIDENT WAVE + REFLECTED WAVE = STANDING WAVE



VSWR
 $\frac{AB}{CD} = 3.00$

Smith Chart, Impedance and Admittance

Move Gm (Gamma) on the Smith Chart.
(created 3/21/03, last modified 12/03/20)

Gm = Complex Reflection Coefficient

Gm relates reflected voltage to incident voltage.

Gm: (0.22, 63.43°)

Gm_real = 0.100

Gm_imaginary = 0.200

Normalized Impedance = $Z_n = (1 + G_m) / (1 - G_m) = R + jX = \text{Resistance} + j\text{Reactance}$

Zn relates voltage to current.

Resistance R (ohms) = 1.118

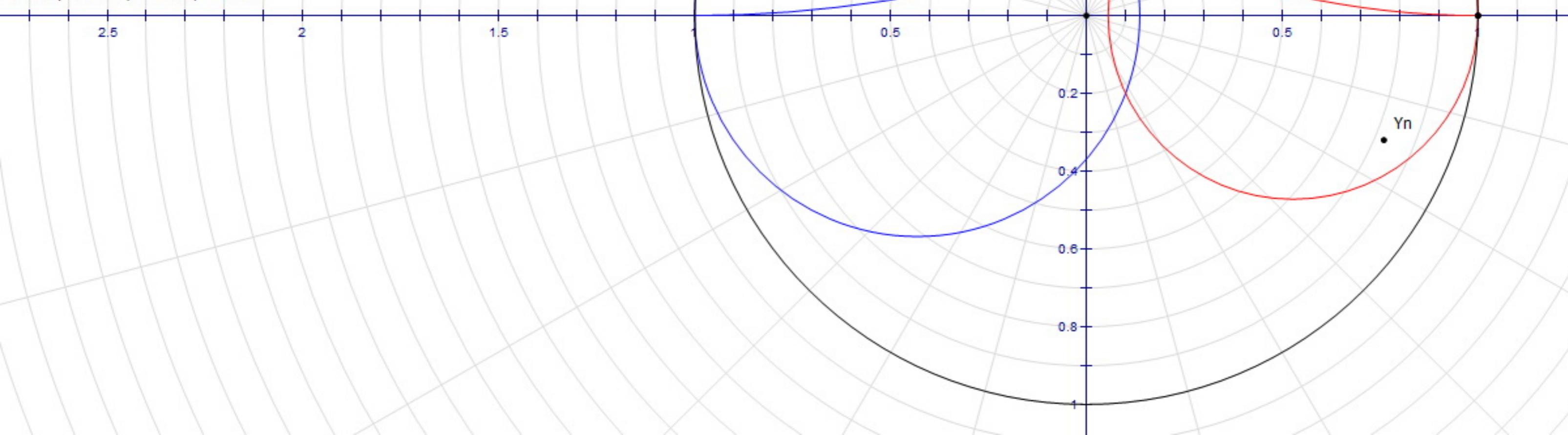
Reactance X (ohms) = 0.471

Normalized Admittance = $Y_n = 1 / Z_n = G + jB = \text{Conductance} + j\text{Susceptance}$

Yn relates current to voltage.

Conductance G (siemens) = 0.760

Susceptance B (siemens) = -0.320



RF Demo Kit

for NanoVNA-F WWW.SYSJOINT.COM

1

Smith chart plot showing a point on the right side of the chart, indicating a purely resistive load.

Physical component of a parallel LC circuit on a PCB.

2

Smith chart plot showing a point on the left side of the chart, indicating a purely inductive load.

Physical component of a series LC circuit on a PCB.

3 33 OHM
SWR=1.5

Smith chart plot showing a point on the right side of the chart, indicating a purely resistive load.

Physical component of a 33 Ohm resistor on a PCB.

4 75 OHM
SWR=1.5

Smith chart plot showing a point on the right side of the chart, indicating a purely resistive load.

Physical component of a 75 Ohm resistor on a PCB.

5 BSF 6.5MHz

Graph showing a bandpass filter response with a sharp peak at 6.5 MHz.

Physical component of a 6.5 MHz bandpass filter on a PCB.

6 BPF 10.7MHz

Graph showing a bandpass filter response with a sharp peak at 10.7 MHz.

Physical component of a 10.7 MHz bandpass filter on a PCB.

7

Smith chart plot showing a point on the left side of the chart, indicating a purely inductive load.

Physical component of a series LC circuit on a PCB.

8

Smith chart plot showing a point on the right side of the chart, indicating a purely resistive load.

Physical component of a parallel LC circuit on a PCB.

9

Smith chart plot showing a point on the left side of the chart, indicating a purely inductive load.

Physical component of a series LC circuit on a PCB.

10

Smith chart plot showing a point on the right side of the chart, indicating a purely resistive load.

Physical component of a parallel LC circuit on a PCB.

11 LPF 400MHz

Graph showing a lowpass filter response with a sharp drop at 400 MHz.

Physical component of a 400 MHz lowpass filter on a PCB.

12 HPF 500MHz

Graph showing a highpass filter response with a sharp rise at 500 MHz.

Physical component of a 500 MHz highpass filter on a PCB.

13 Short

Smith chart plot showing a point on the left side of the chart, indicating a purely inductive load.

Physical component of a short circuit on a PCB.

14 Open

Smith chart plot showing a point on the right side of the chart, indicating a purely resistive load.

Physical component of an open circuit on a PCB.

15 Load

Smith chart plot showing a point on the right side of the chart, indicating a purely resistive load.

Physical component of a load on a PCB.

16 Thru

Smith chart plot showing a point on the right side of the chart, indicating a purely resistive load.

Physical component of a thru connection on a PCB.

17 ATT -10dB

Graph showing an attenuator response with a sharp drop at -10 dB.

Physical component of a -10 dB attenuator on a PCB.

18 ATT -3dB

Graph showing an attenuator response with a sharp drop at -3 dB.

Physical component of a -3 dB attenuator on a PCB.

Smith Chart Calc



Z0: 50Ω Frequency: 147.0MHz

Start: 25.0 + j60.0Ω Target: 50.0 + j0.0Ω

Final: 10.88 + j12.31Ω ρ: 0.66

Series Capacitor 65.07pF



Shunt Capacitor 7.83pF



Series Inductor 53.49nH



Shunt Inductor 34.18nH



Series Resistor 40.53Ω



Shunt Resistor 44.56Ω

